Exercice 1 (10pts) True or false, justify 
2 points for each correct answer. Each incorrect or unjustified response removes 1 point.

1. If \( \lim_{x \to 0^+} f(x) = +1 \) et \( \lim_{x \to 0^-} f(x) = -1 \) then \( f \) continuous at \( x = 0 \).

2. If \( u = 0 \) \( \Rightarrow \) \( v = 0 \) then \( \frac{u}{v} = \frac{u^2}{v^2} \).

3. If \( f \) is a \( T \) periodic function then \( f(x + T) = f(x) = f(x - T) \).

4. If \( \lim_{x \to +\infty} \frac{x}{1^x} = +\infty \) then \( \lim_{x \to +\infty} \frac{x^2}{2^x} = +\infty \).

5. The Rolle’s theorem is applicable to the function \( f(x) = x^{2/3} \) in \( [8, 8] \).

Exercice 2 (3 \times 5 = 15pts) : Independent questions :

1. If \( y(x) = x^2 e^x \) Prove that \( \frac{d^n}{dx^n} y = (x^2 + 2nx + n(n-1)) e^x \).

2. Let \( f: \mathbb{R} \to \mathbb{R} \), continuous, such as, \( \forall x \in \mathbb{R} \) and \( \forall y \in \mathbb{R} \), \( f(x + y) = f(x) + f(y) \). Show that \( \forall n \in \mathbb{N} : f(n) = nf(1) \).

3. Let \( (u_n)_{n \in \mathbb{N}} \) a numerical sequence such that : \( u_n = \frac{\ln(n)}{\sqrt{n^4 + 4}} \). Show that \( u_n \) is convergent and calculate its limit.

4. Calculate \( \lim_{x \to +\infty} \left( \sqrt{x} - \sqrt{x + 1} \right) \).

5. Calculate the derivative of \( g(x) = \sqrt{x^2 + 2\sqrt{x}} \).

Exercice 3 (10pts) Verify Lagrange’s mean value theorem for the function \( f(x) = (x - 1)(x - 2)(x - 3) \) in \( [0, 4] \).

Exercice 4 (20pts) Let \( (u_n) \) be the sequence defined recursively by

\[
\begin{align*}
\frac{u_{n+1}}{2} &= u_n + 3 \\
and u_0 &= 1
\end{align*}
\]

1. Let \( (v_n) \) the sequence defined by \( v_n = u_{n+1} - u_n \). Show that \( v_n \) is a geometric sequence. Write \( v_n \) as function of \( n \).
2. Let \((S_n)\) the sequence defined by : \(S_n = \sum_{k=0}^{n-1} v_k\). Find the limite of \(S_n\).

3. Deduce that \(u_n\) is convergent and calculate its limit.

**Exercice 5 (30 pts)** Consider the function :

\[
f(x) = (x^2 - 1) \ln \left| \frac{x + 1}{x - 1} \right|
\]

1. Determine the domain of definition of \(f\) and show that it is odd.

2. Evaluate : \(\lim_{x \to 1} f(x)\), deduce \(\lim_{x \to -1} f(x)\)

3. Deduce the function \(g(x)\), the extension by continuity of \(f\). The function \(g\) it is differentiable at \(x = 1\) ?

4. Find the equation of the asymptote at the curve of \(f\)

5. Let \(h(x) = \ln \left| \frac{1 + x}{1 - x} \right| - \frac{1}{x}\) a function defined for any \(x \in D_1 = ]0,1[ \cup ]1, +\infty[\)

   (a) By studying the function \(h\), Show that \(h(x) > 0\) for any \(x \in ]1, +\infty[\) and there \(\alpha \in ]0,1[\) sush that \(h(x) < 0\) if \(x \in ]0,\alpha[^\) and \(h(x) > 0\) if \(x \in ]\alpha, 1[\)

   (b) Show that for all \(x \in D_1\) we have \(f'(x) = 2xh(x)\).

**Bonus (5 pts)** Draw the table of variations of \(f\). Calculate \(f'(0)\) and plot the curve of \(f\).

**Exercice 6 (15 pts)** Find the dimensions of a rectangle with maximum area that we can write inside a circle of radius \(R\).